Monitoring concrete strength by the cusum system

1. Introduction

The production of concrete must be controlled in such a way that strength specifications are met and materials costs are kept as low as possible. The cusum system is a practical means of meeting these requirements.

In general, the cusum system measures performance relative to design intentions. It compares results with target values and checks whether they are consistent with intended and required levels.

The cusum system is used for monitoring trends in mean strength, standard deviation and the relationship between early-age and 28-day strengths. It assists detection of changes in these properties, and indicates when action should be taken to increase the probability of meeting the specification or to reduce the materials cost of the concrete.

When used to monitor concrete strength, the cusum system has advantages over other systems:

- The cusum system is more sensitive in detecting changes of the magnitude experienced with concrete production.
- Reliable decisions can be made on fewer results.
- The trend of results can be identified from the general slope of a graph.
- The slopes of graphs can be used to determine the magnitudes of properties (i.e. mean strength and standard deviation).
- Positions of changes in the slopes of graphs indicate approximately when changes occurred.

Against these advantages there may be a slightly increased complexity in processing data compared with other systems. This is of little consequence when the system is computerized.

The aim of this publication is to provide the knowledge needed to monitor concrete strength by the cusum system. Principles of the cusum system and preparatory work are dealt with. The operation of a cusum system to monitor concrete strength is then covered.

This publication deals mainly with monitoring the strength of a single grade of concrete (using a single set of materials) by the cusum system. Combining (or massing) results from several mixes into one cusum is dealt with briefly.

2. Principle of the cusum system

The essential principle is that differences between results and their target values are calculated and added cumulatively to form a cumulative sum (cusum). When this cusum is plotted graphically against the sequence of results, a visual presentation of the trend relative to the target level is produced.

3. Applications

3.1 Monitoring mean strength

To monitor mean strength, 28-day strengths are either determined or predicted from early-age strengths. The target mean strength (TMS) is subtracted from each result to obtain a difference. As results become available, the differences are cumulatively summed to form the cusum M. (See Table 1.) A positive difference indicates that the result in question is greater than TMS. A negative difference indicates that the result is lower than TMS. If the average strength is greater than TMS, then the slope of a plot of cusum M vs result number will be positive, or upward to the right. (See Figure 1.) Similarly, a negative or downward slope indicates that the mean strength is below TMS.

Table 1: Example of cusum applied to mean strength when TMS = 38 MPa

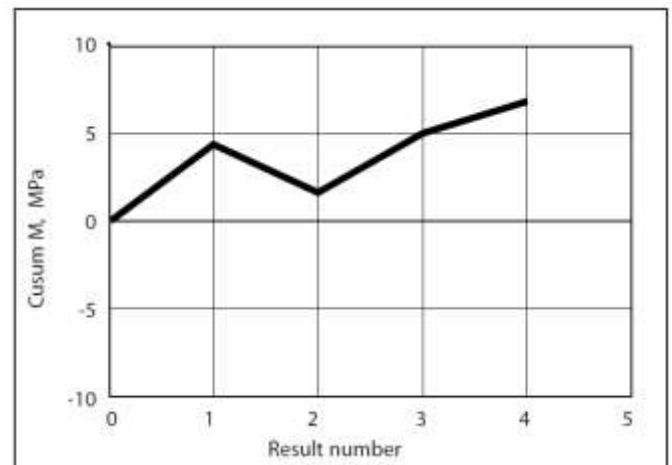| Result no.	28-day strength MPa	Difference from TMS (TMS = 38 MPa) MPa	Cusum M MPa
1	42	+4	4
2	36	-2	2
3	41	+3	5
4	40	+2	7

Figure 1: Cusum plot of mean strength data from Table 1

3.2 Monitoring standard deviation

When applying cusum techniques to the standard deviation (SD) of concrete strengths, use is made of the relationship between SD and the range of successive pairs of results (see Table 2 and Figure 2). (For our purposes, range can be defined as the absolute value of the difference between successive results.)

It can be shown statistically that the mean range of successive pairs of a large number of results approximates to the SD of those results multiplied by 1,128.

Thus, the target mean range = 1,128 x target SD. A cusum R chart can then be plotted for the difference between the actual range and the target mean range (TMR). Upward slopes indicate SD greater than target mean, whilst downward slopes indicate SD lower than target. To simplify the calculation, the target range is normally rounded to the nearest 0,5 MPa unless computers are used. See Table 3 for typical values of TMR.

Table 2: Example of cusum applied to standard deviation when target mean range (TMR) = 6 MPa

Result no.	28-day strength MPa	Range of pair of results MPa	Range minus TMR MPa	Cusum R (cusum of differences) MPa
1	42	-	-	-
2	36	6	0	0
3	41	5	-1	-1
4	40	1	-5	-6

3.3 Monitoring correlation between predicted and actual strengths

Normally, concrete strength at 28 days is specified and a cusum system can be used to monitor 28-day results as they become available. However, the producer may wish to detect changes in concrete strength performance earlier than is possible using this method. A response more rapid than that possible from 28-day results can be obtained using predictions from earlier age tests.

The predicted 28-day strength can then be used in the CUSUM calculations and subsequently confirmed, or the prediction modified, on the basis of actual 28-day results received at a later date.
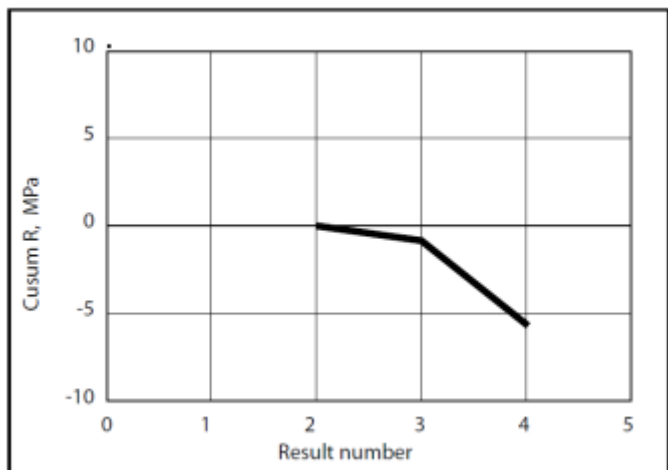

Figure 2: Plot of cusum r from Table 2

Table 3: Target mean ranges (rounded) for common values of standard deviation

Standard deviation MPa	Target mean range rounded, MPa
3,5	4,0
4,0	4,5
4,5	5,0
5,0	5,5
5,5	6,0
6,0	7,0
6,5	7,5
7,0	8,0

Predicted strengths are commonly estimated from the strengths of cubes cured by the standard method for seven days, but predictions based on accelerated tests may be used.

The relationship between actual and predicted 28-day strength is referred to as correlation. The relationship is often stable for considerable periods, but in the longer term changes will occur as a result of variations in cement and other raw material properties.

A cusum system can detect changes by monitoring the correlation difference (actual minus predicted 28-day strength). If the difference is positive, the prediction system is underestimating; if negative, it is overestimating. The cumulative sum of these differences is called cusum C. Thus an upward slope of the plot of cusum C represents underestimation, and a downward slope indicates overestimation. Table 4 and Figure 3 illustrate how correlation difference is monitored by the cusum system.

Table 4: Example of cusum applied to correlation of predicted and actual 28-day strengths

Result no.	Early result MPa	Predicted 28-day strength MPa	Actual 28-day result MPa	Correlation difference MPa	Cusum C MPa
1	43,0	54,0	53,0	-1,0	-1,0
2	28,5	38,0	35,0	-3,0	-4,0
3	32,0	42,0	41,0	-1,0	-5,0
4	29,0	38,5	38,5	0,0	-5,0

4. Preparatory work

Preparatory work needed before concrete strength can be monitored by the cusum system involves the following steps:

1. Arrange to obtain valid cube strength results. Standard methods should be followed for the making, curing and testing of cubes. There is no point in applying the cusum system to unreliable test results.
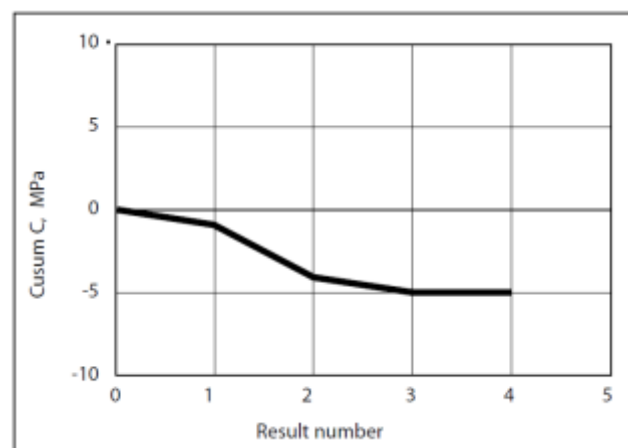

Figure 3: Plot of cusum C from Table 4

2. Choose an achievable target value for the SD of results. Where sufficient valid test results are available from previous production with the materials, staff and production facilities to be used, a suitable estimate of SD achieved may be calculated from this information. If appropriate results are not available, then Table 5 can be used to estimate the expected SD.

Only skilled and experienced teams are considered able to maintain an SD better than 5 M Pa and hence values below 5 M Pa should be chosen only after careful consideration.

Table 5: Standard deviation for different degrees of control

Degree of control	Standard deviation MPa
Excellent	2,5
Good	4,0
Average	5,5
Poor	7,0
Very poor	8,5

3. Choose a suitable target mean strength (TMS). For an existing plant where strength performance has been satisfactory, the average strength of recent production for a mix may be used for TMS for that mix. For a new plant, the selection of TMS justifies careful thought and the following factors should be considered:

- In South African design and construction practice, the specified strength or grade of a mix is the strength which must be exceeded by at least 95% of production. This is termed the characteristic strength (CS) of the mix. For normally distributed strength results this condition requires TMS to exceed the CS by not less than 1,64 times the SD of the population.

- For new production facilities, certain overseas structural codes provide guidelines for the initial strength margins to be used between TMS and CS.

- Although by definition CS implies no more than 5% of test results falling below the CS, certain code provisions make it expedient to set target margins greater than 1,64 x SD for the difference between TMS and CS to reduce the percentage of results falling below CS.

4. Establish a correlation between early-age strength and 28-day strength. The strengths of cubes cured by the standard method for seven days are often used to predict 28-day strengths. The initial correlation between early and 28-day strengths can be established by either of two methods:

- Use existing valid test results obtained from concrete made of materials to be used.

- Make and cure cubes of laboratory or production mixes containing cement and aggregate to be used and determine early and 28-day strengths.

Correlations between early and 28-day strengths can be presented as:

- Factors
- Graphs of early strength vs 28-day strength
- Tables

If only one grade of concrete is to be monitored, then a factor can be determined for use in predicting 28-day strengths from early-age strengths.

A specific correlation will apply to one source and type of cement, and a finite period. It should not be affected by minor changes of aggregate type, but extreme changes could be significant (e.g. from andesite to quartzite).

Separate correlations should be established for:

- Mixes containing different admixtures
- Mixes containing different binders

Table 6: Typical cusum calculations (modified from reference 1)

Initial plant parameters
Standard deviation 6 MPa (Target mean range = 7 MPa)
Target mean strength 42 MPa

Cube ref.	Date cast	Early result (accel-erated) MPa	Est. 28-day strength MPa	4 minus target mean strength MPa	Cusum M Cumulative sum of 5 (for mean strength control chart) MPa	Range of pairs of values of 4 MPa	7 minus target mean range MPa	Cusum R Cumulative sum of 8 (for standard deviation control chart) MPa	Actual 28-day result MPa	Correlation difference: 10 minus 4 MPa	Cusum C Cumulative sum of 11 (for correlation control chart) MPa
1	2	3	4	5	6	7	8	9	10	11	12
1	Feb 5	43,0	54,0	+ 12,0	+ 12,0				53,0	− 1,0	− 1,0
2	8	28,5	38,0	− 4,0	+ 8,0	16,0	+ 9,0	+ 9,0	35,0	− 3,0	− 4,0
3	9	32,0	42,0	0	+ 8,0	4,0	− 3,0	+ 6,0	41,0	− 1,0	− 5,0
4	9	29,0	38,5	− 3,5	+ 4,5	3,5	− 3,5	+ 2,5	38,5	0	− 5,0
5	10	33,5	43,5	+ 1,5	+ 6,0	5,0	− 2,0	+ 0,5	43,5	0	− 5,0
6	11	31,0	41,0	− 1,0	+ 5,0	2,5	− 4,5	− 4,0	39,0	− 2,0	− 7,0
7	12	33,0	43,0	+ 1,0	+ 6,0	2,0	− 5,0	− 9,0	47,5	+ 4,5	− 2,5
8	15	25,0	34,0	− 8,0	− 2,0	9,0	+ 2,0	− 7,0	37,5	+ 3,5	+ 1,0
9	16	42,0	53,0	+ 11,0	+ 9,0	19,0	+ 12,0	+ 5,0	53,5	+ 0,5	+ 1,5
10	17	24,5	33,5	− 8,5	+ 0,5	19,5	+ 12,5	+ 17,5	32,5	− 1,0	+ 0,5
11	17	25,0	34,0	− 8,0	− 7,5	0,5	− 6,5	+ 11,0	30,0	− 4,0	− 3,5
12	19	33,5	43,5	+ 1,5	− 6,0	9,5	+ 2,5	+ 13,5	43,5	0	− 3,5
13	22	32,0	42,0	0	− 6,0	1,5	− 5,5	+ 8,0	40,5	− 1,5	− 5,0
14	23	24,5	33,5	− 8,5	− 14,5	8,5	+ 1,5	+ 9,5	34,0	+ 0,5	− 4,5
15	24	28,5	38,0	− 4,0	− 18,5	4,5	− 2,5	+ 7,0	35,5	− 2,5	− 7,0
16	25	33,0	43,0	+ 1,0	− 17,5	5,0	− 2,0	+ 5,0	41,5	− 1,5	− 8,5
17	26	30,0	39,5	− 2,5	− 20,0	3,5	− 3,5	+ 1,5	38,5	− 1,0	− 9,5
18	Mar 1	34,0	44,0	+ 2,0	− 18,0	4,5	− 2,5	− 1,0	46,5	+ 2,5	− 7,0
19	2	31,0	40,5	− 1,5	− 19,5	3,5	− 3,5	− 4,5			
20	2	32,0	42,0	0	− 19,5	1,5	− 5,5	− 10,0			
21	3	27,5	37,0	− 5,0	− 24,5	5,0	− 2,0	− 12,0			
22	4	24,5	33,5	− 8,5	− 33,0	3,5	− 3,5	− 15,5			
23	5	32,0	42,0	0	− 33,0	8,5	+ 1,5	− 14,0			
24	8	24,0	32,5	− 9,5	− 42,5	9,5	+ 2,5	− 11,5			
25	9	26,5	35,5	− 6,5	− 49,0	3,0	− 4,0	− 15,5			
26	10	24,0	32,5 (35,0)	− 9,5	− 58,5	3,0	− 4,0	− 19,5			

Cement content increased by 15 kg/m³ to restore mean strength to its target.

27	12	24,5	33,5	− 8,5	− 8,5	1,5	− 5,5	− 25,0			
28	12	38,5	49,0	+ 7,0	− 1,5	15,5	+ 8,5	− 16,5			
29	13	37,5	48,0	+ 6,0	+ 4,5	1,0	− 6,0	− 22,5			
30	13	30,0	39,5	− 2,5	+ 2,0	8,5	+ 1,5	− 21,0			
31	15	26,0	35,0	− 7,0	− 5,0	4,5	− 2,5	− 23,5			
32	17	39,5	50,0	+ 8,0	+ 3,0	15,0	+ 8,0	− 15,5			
33	18	39,0	49,5	+ 7,5	+ 10,5	0,5	− 6,5	− 22,0			
34	19	38,0	48,5	+ 6,5	+ 17,0	1,0	− 6,0	− 28,0			
35	19	32,0	42,0	0	+ 17,0	6,5	− 0,5	− 28,5			
36	22	27,5	36,5	− 5,5	+ 11,5	5,5	− 1,5	− 30,0			
37	23	32,0	42,0	0	+ 11,5	5,5	− 1,5	− 31,5			
38	24	32,0	42,0	0	+ 11,5	0	− 7,0	− 38,5			
39	24	33,0	43,0	+ 1,0	+ 12,5	1,0	− 6,0	− 44,5			
40	25	33,5	43,5	+ 1,5	+ 14,0	0,5	− 6,5	− 51,0			
41	26	37,5	48,0 (45,0)	+ 6,0	+ 20,0	4,5	− 2,5	− 53,5			

Cement content reduced by 20 kg/m³.

42	29	31,5	41,0	+ 2,0	+ 22,0	4,0	− 1,0	− 1,0			

Tabulation of results in Table 6

Columns 1 to 4 Basic data reference number, date cast, early and predicted 28-day cube strengths.

Columns 5 to 6 Cusum calculation for mean strength.

Columns 7 to 9 Cusum calculation for mean range allied to standard deviation.

Columns 10 to 12 Actual 28-day cube strengths and the cusum of the correlation difference.

Note on range calculations: Caution is necessary when calculating the range just after a change has been made in cement content. Since it is the variability of the general production control that is important, the result just before the change should be adjusted to a value appropriate to the new cement content before calculating the next range. This is shown in Table 6 by the figures in brackets (results 26 and 41).

5. Operating a cusum system to monitor concrete strength

Concrete cube results are obtained and processed on a daily basis. Processing involves calculating, plotting and assessing trends as described below. Occasionally unusual results may occur. A discussion on how to handle these is given in Appendix A.

5.1 Calculations

Calculations are best carried out in tabular form (or by computer) and will be explained by referring to Table 6, which is taken from reference 1.

As each early-age result is received, enter it in the table with its identifying reference data.

Consider cube reference 5 in Table 6 relating to concrete mixed on 10 February. Determine the predicted 28-day result from the correlation system and insert it in column 4.

Deduct the target mean strength and enter the difference in column 5 (43,5 - 42,0 = + 1,5). Add this difference to the previous cusum M in column 6 (4,5 + 1,5 = 6,0) and enter the new cusum M in column 6.

Calculate the range between predicted (or estimated) 28-day strengths of cubes 5 and 4 (43,5 - 38,5 = 5,0 and insert it in column 7. Deduct the target mean range (5,0 - 7,0 = - 2,0) and insert the difference in column 8. Add this to the previous cusum R (2,5 + (- 2,0) = + 0,5) and enter the new cusum R in column 9.

Enter actual 28-day results in column 10 as they become available. Deduct the predicted 28-day strength (43,5 - 43,5 = 0) and enter the difference in column 11. Add this to the previous cusum C (- 5,0 + 0 = - 5,0) and enter the new cusum C in column 12.

5.2 Plotting cusum charts

One of the advantages of the cusum system is that trends indicating deviations from target and/or changes in performance can be readily identified from a graphical presentation.

The presentation can be enhanced or diminished by adopting different scales for the graph. For example, the slope will increase if either the vertical scale for the cusum axis is increased or the horizontal scale for result numbers is reduced. The selection of correct scales is thus very important to ensure suitable presentation. A useful approach is to establish a convenient distance on the horizontal axis between successive plotting positions (say 5 mm for a chart for desk use or 10 mm for a chart for wall display), then use this distance on the vertical cusum axis to represent between one and two standard deviations. Thus, depending on the expected standard deviation and the use of the chart, 5 mm (or 10 mm) could represent 5 or 10 MPa on the vertical cusum scale.

Cusum data are plotted separately on the vertical axis against the result number on the horizontal axis. The cusum data for mean, range and correlation from columns 6, 9 and 12 of Table 6 are shown on cusum charts in Figure 4.

5.3 Assessing trends

If a segment of a cusum graph is horizontal, the property being plotted over that period is the same as the target value. A slope indicates that the property is different to its target value.

Figure 4 raises questions: Are the observed trends significant? Are they large enough to require corrective action? A method has been devised to determine whether a significant change has occurred. After each point has been plotted, a transparent mask in the shape of a truncated v turned through 90° (see Figure 5) is placed over the plot, with the "lead point" (see Figure 5 and 6) over the last cusum result plotted. If the plot remains inside the boundaries of the truncated v and its extension, no significant change has occurred. However, if the plot crosses the truncated V or its extension, a significant trend is deemed to have occurred and a requirement for action is indicated. This mask is applied to the plot each time a new result is added, and a further check is made. (The use of masks detecting significant changes in cusum R and cusum M is shown on Figure 7 by dotted lines.)

Design of masks

The geometry of the mask is linked to statistical probabilities. Significant changes must be detected as soon as possible, but the system must not be over-sensitive so that it responds to insignificant changes.
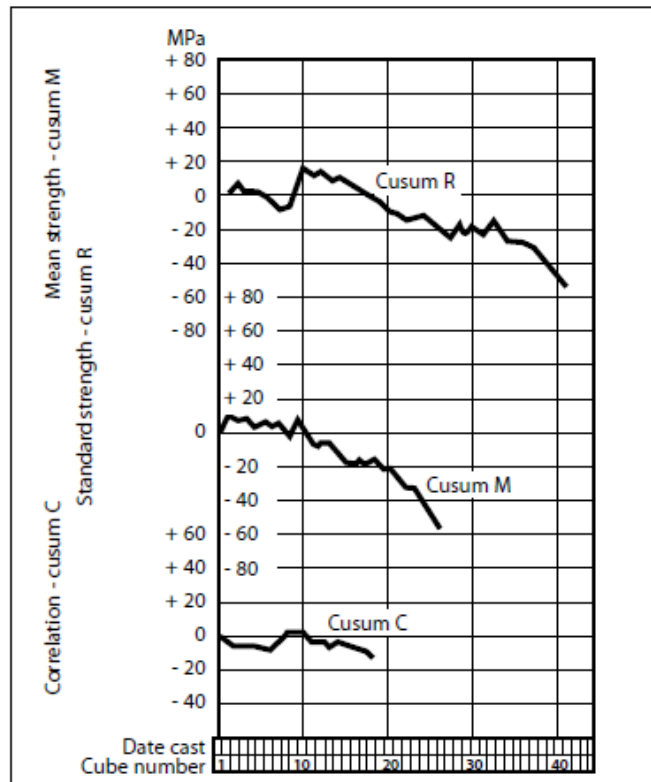

Figure 4: Cusum charts for standard deviation, mean strength and correlation (modified from reference 1)
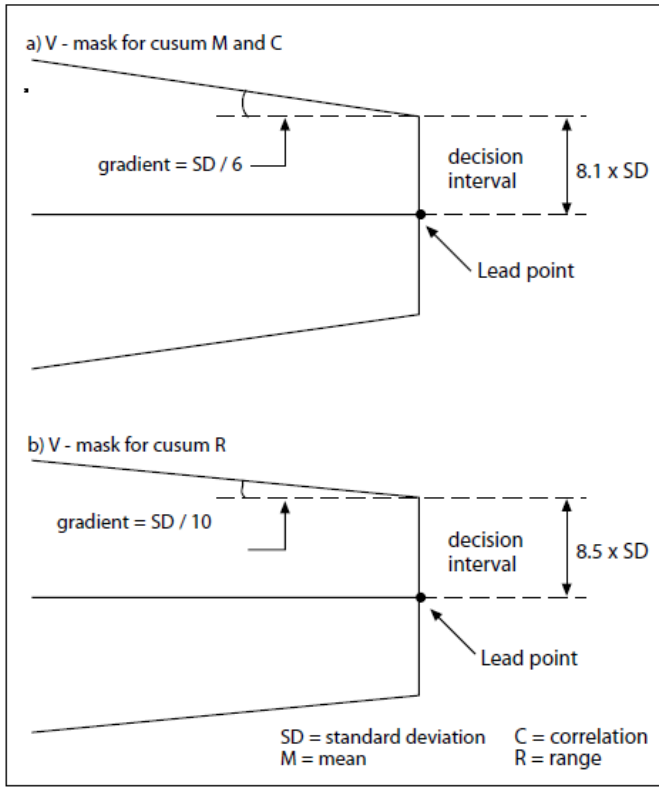
Figure 5: v-mask design for concrete cusum (modified from reference 1)

Masks can be designed using either computer simulation techniques or nomograms available in BS 5703: Part 3. There is no absolute solution, the design being a compromise between the confidence level and how rapidly detection is required. The masks from one system are shown in Figure 5, which shows mask geometry to be dependent on the target SD which, in turn, requires preparation of individual masks for each value of the SD. Alternatively these individual masks can be superimposed on each other to form a multiple mask as shown in Figure 6.
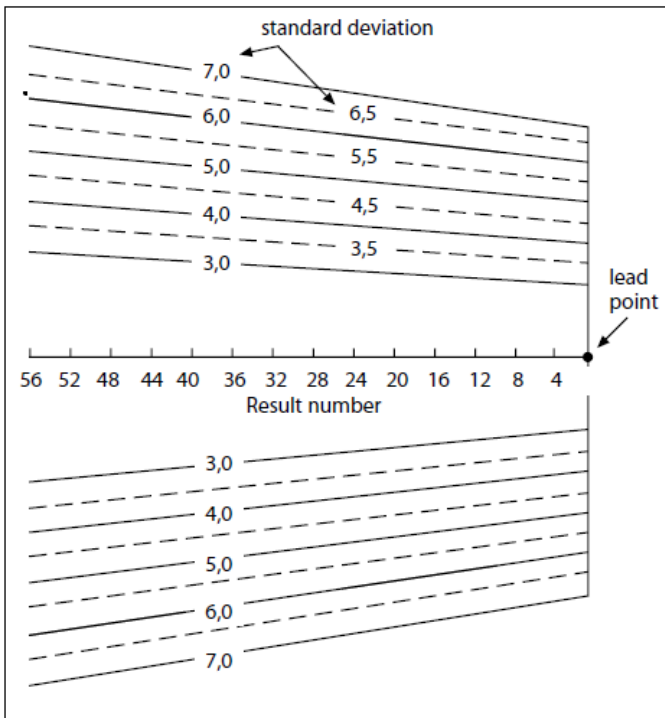
Figure 6: Multiple v-mask for cusum m (modified from reference 1) (Similar masks can be produced for cusum R and cusum C.)

It is recommended that masks be made long enough to include 60 result numbers on the horizontal axis. If this is done, then when a significant change has occurred, the cusum plot will cross the limbs of the v-mask; if the plot crosses the extension of a limb, then a change requiring an insignificant change to cement content is indicated.

The SD used for producing masks for detecting changes in correlation is much smaller than the SD used on masks for cusum M and R. Typically, an SD of 2,5 MPa is adopted. However, if unusually good conditions exist, a more sensitive mask based on 2 M Pa can be used. Exceptionally, in conditions where perhaps different sources of cement are in use for the same supply of concrete, a value of 3,25 MPa could be adopted. Caution is necessary, in this latter case, to ensure that the necessity for a high value is not the result of poor testing.

Use of masks
Each chart is examined as every new result is plotted, with the lead point of the appropriate v-mask placed on the new point.

Consider the plot of cusum M, in Figure 4, when the transparent mask for mean strength is placed on the graph. No change is indicated for the first 25 results (i.e. the plot does not cross the limbs of the v). After result 26 is plotted a change is indicated. By observing the slope of the plot it is apparent that a change occurred on or about result 10. The system is indicating that a significant change in mean strength has occurred and that action should be taken to correct the observed difference between target mean strength and predicted 28-day strength. (See Figure 7.)
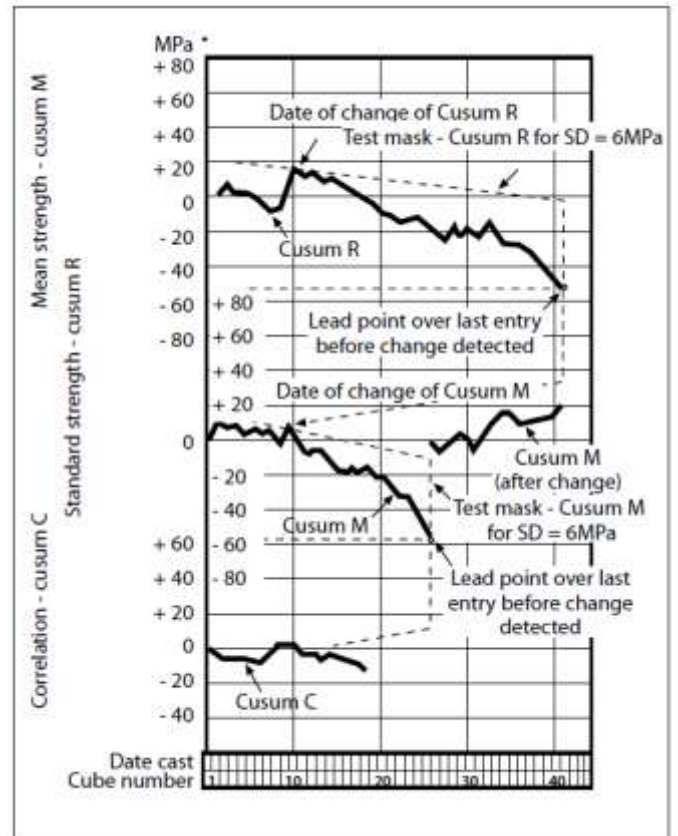

Figure 7: Cusum charts for standard deviation, mean strength and correlation (modified from reference 1)

In similar fashion, but using the appropriate mask for cusum R, a significant change in SD is indicated at result 41, originating at about result 10. As a significantly lower SD is indicated, the producer could use this to recalculate his target mean strength and so save cement.

Cusum R is examined first, i.e. before cusum m or cusum C, for reasons which will be explained later.

The system of predicting 28-day strengths is checked when the appropriate v-mask is used, in similar fashion, on the cusum C chart. (Remember that a v-mask with SD within the range 2-3,25 M Pa is normally used on cusum C.)

5.4 Action following indicated changes
When significant changes in mean strength, SD or correlation are indicated, action may be taken. There are several interactions between the three cusums and these need to be considered when changes are made.

Change indicated in cusum M
When the indicated change is in mean strength, the change in mean strength must be determined and the cement content altered to restore the mean strength to the TMS. This will be illustrated by means of the following example:

Suppose a plant, at which it is known that a change of 8 kg/m^3 of cement produces a change in mean strength of 1 MPa, produces Grade 30 concrete with TMS = 42 MPa and cement content = 400 kg/m^3. After plotting result 26 on the cusum graphs for Grade 30 concrete, the cusum M graph cuts the v-mask at result 9. From the table of calculations, cusum M at results 9 and 26 is + 9,0 and - 58,5 MPa respectively.

The average departure from target over a segment of a cusum graph equals change in cusum divided by the number of values in the segment. When a change is indicated the segment to consider is that between the point on which the lead point of the v-mask has been placed and the point which cuts the limb of the v-mask. In the example,

> change in cusum = - 58,5 - (+ 9,0) = - 67,5 MPa
> number of values = 26 - 9 = 17
> Therefore, average departure from target
> = - 67,5 ÷ 17 = - 3,97 — say – 4 MPa

Over the segment, therefore, the mean strength has been approximately 42 MPa – 4 MPa = 38 MPa instead of TMS (= 42 MPa); and to restore the mean strength to TMS, it needs to be raised by 4 MPa.

From the information given, we know that an increase in cement content of 8 kg/m^3 raises mean strength by about 1 MPa.

Therefore, the change in cement content needed to raise mean strength by 4 M Pa is 4 x 8 = 32 kg/m^3.

Usually, an anti-hunting factor between 0,5 and 1,0 is applied to the change to reduce the probability of over-reaction. Assuming an anti-hunting factor of 0,75, the change becomes 32 x 0,75 = 24 kg/m^3. This figure may be rounded, if so desired.

The change in cement content is assumed to have restored the mean strength to the target value and no adjustment to the cusum R or cusum C chart is necessary. However, after a change, the cusum m plot is restarted, normally from zero, and all results prior to the change are ignored in subsequent analyses of mean strength. For the result at which a change of mix is imposed, a revised value of estimated strength should be calculated, based on the new cement content, for use in calculating the next range value. The cusum R plot is continued without adjustment and results prior to the change in mean strength (back to the last change in SD) are included in subsequent analyses. (These changes are shown at result 26 in Table 6 and Figure 7. In Table 6 a different change was made to the cement content because of the difference in the average strength performance of cements from the UK [where the original example was produced] and the RSA.)

Change indicated in cusum R
The size and shape of v-masks depend on the SD in use. Accordingly, as new results are entered in the cusum, the cusum R is examined first. When a change in SD is detected, new masks are adopted for both the future cusum R and the current cusum M plots. The mask appropriate to the new value of SD is used to assess any additional or subsequent action arising on the cusum m. Provided the correct action is taken to adjust the cement content for the change detected, it is not necessary to restart the cusum M.

Change in SD is calculated from: change in cusum r divided by number of values in the segment between the last point on which the lead point of the v-mask was placed and the point which cuts the truncated v. (This calculation is approximate.)

The change in SD is used to calculate the current SD. The current SD determines which v-masks to use.

The current SD is used to calculate a new TMS. The new and the old TMS values are used to calculate an appropriate change to TMS which is then used to calculate a change to cement content. (The calculations required are shown in Appendix B.)

After such a change, the cusum r plot is restarted, normally from zero, and all results prior to the change are ignored in subsequent analyses of standard deviation. The cusum M plot is continued without adjustment and results previous to the change in SD (back to the last change in mean) are included in subsequent analyses.

The masks for the new level of SD are used for further analyses of both plots and, in the case of cusum M, an immediate check is made with the new mask on previous results to determine any change which might have occurred.

Indicated change in cusum C

When a change in correlation is indicated, the following should be done.
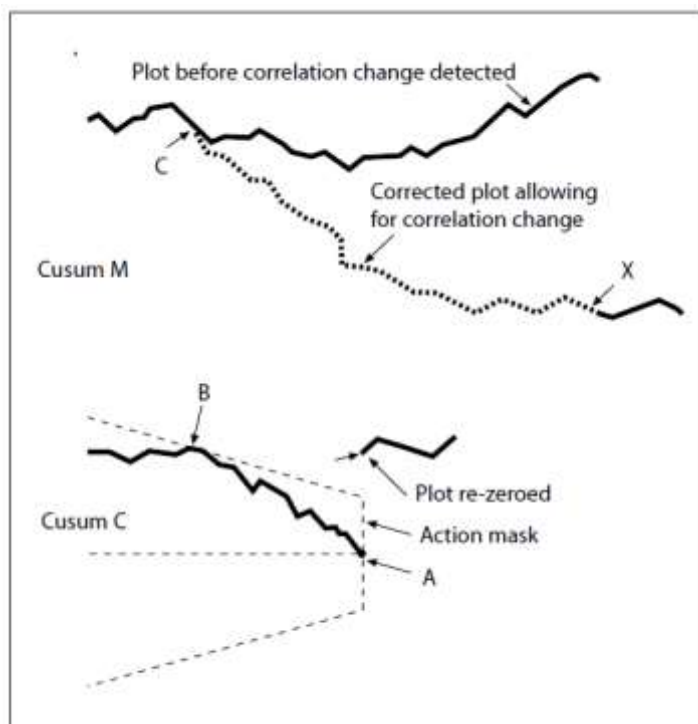

Figure 8: Interaction due to change in correlation (modified from reference 1) (The corrected plot of cusum m should be examined with the mask for cusum m at point X before adding any new results.)

A new correlation relationship must be determined. It is suggested that the early and 28-day strengths, in the segment between the point on which the lead point of the v-mask was placed and the point which crosses the v-mask, be used to produce a new factor, graph or table for predicting 28-day strengths.

When a change is indicated, the mean strengths have either been under- or over-estimated. The cusum M could be significantly adrift and need re-calculation. Thus the results subsequent to the indicated point of change should be replotted on the cusum M, based on predictions of 28-day strength from the new correlation.

An immediate check should be made on cusum M with the appropriate mask before adding any new results. (See Figure 8.) If a previous change in mean strength is indicated, as a result of using the corrected strength values, this must be made before continuing the analysis.

The process described above is illustrated in Figure 8.

When an action mask is placed over the plot of cusum c with the lead point at A, the plot cuts the arm of the V at B. A new correlation must be determined and used to predict 28-day strengths for all points on cusum m between C and X. (C is the point on cusum m which corresponds to B on cusum c.) cusum M is replotted from C to X. The corrected plot of cusum M should be examined with the relevant mask at point X before adding any new results. If this check indicates no change in mean strength, then the plot of cusum m continues from X.

Cusum R is relatively unaffected, except for a small effect for the first result when the correlation originally changed, and it is generally not necessary to re-plot cusum R.

The cusum c plot is restarted, normally from zero.

6. Cusum using a range of mixes

Cusum methods have been described above as they would be applied to the strength of one grade of concrete. In many situations concrete plants may be producing a range of mixes. The results of a variety of mixes can be incorporated into a single cusum provided adjustments are applied. For example, consider a cusum running on a mix with TMS of 35 MPa at 28 days, then results from a mix with TMS of 40 M Pa at 28 days could be included provided 5 MPa is deducted before analysis. This assumes that the relationship between TMS and cement content is known.

This approach can be extended to mixes made with different aggregate sizes and slumps, provided the relationships with the standard mix are known.

Reference

1. Brown, B.V. Monitoring concrete by the cusum method, London: Concrete Society, 1984. (Concrete Society Digest no. 6).

Appendix A
Abnormal results

Occasionally unusual results may occur. Such results may or may not be significant in terms of the concrete placed in a structure (depending on the cause), but, since they are abnormal, should not be used in identifying general quality control trends. For example, any result greater than 3 x target SD above or below the mean strength should be rejected and not used in the analysis. If the next result is greater than 2 x target SD from the mean in the same direction, both this result and the previous rejected result should be incorporated, and an immediate investigation made to determine if a serious quality change has occurred. A similar approach can be used for correlation differences, i.e. normally differences greater than 7,5 M Pa are rejected. However, in order to keep cusum c synchronized with the other cusums, it is normal, in such a case, to substitute a "dummy" result based on the average differences of the previous results (e.g. average of the last four values).

Appendix B
Example of calculations needed when a change in cusum R is indicated

Figure 7 shows that after result 41 has been plotted on cusum r, the plot of cusum r cuts the upper limb of the v-mask at result 10. What change in cement content is appropriate?

(In this example TMS = CS + 2 x SD = 30 + 2 x 6 = 42 MPa.)

Difference between TMR and actual average range
= change in cusum R ÷ number of values in segment
= (- 53,5 - 17,5) ÷ (41 - 10)
= - 2,3 MPa

Difference between target SD and current SD
= change in range ÷ 1,128 = - 2,3 ÷ 1,128
= - 2,0 MPa
. . current SD
= target SD + change
= 6 + (- 2,0) MPa
= 4,0 MPa

New TMS
= CS + 2 x SD
= 30 + 2 x 4,0
= 38 MPa

Change in TMS
= new TMS - old TMS
= 38 - 42 MPa
= - 4 MPa

Assume that an increase in cement content of 8 kg/m^3 raises mean strength by about 1 MPa.

Therefore (using an anti-hunting factor of 0,75), change in cement content needed = 0,75 x 8 x (- 4) kg/m3 = - 24 kg/m^3

Masks for the new level of SD (4 MPa) are used for further analyses of both cusum M and cusum R.